CNT

Part - 2

$8 = 2I_1 - 6I_2 + 6I_3$ —— (1)

$I_1 - I_3 = 3A$ —— (2)

writing KVL at mesh 2

$-6I_2 + 4(I_3 - I_2) + 2(I_1 - I_2) = 0$

$2I_1 - 12I_2 + 4I_3 = 0$

$I_1 - 6I_2 + 2I_3 = 0$ —— (3)

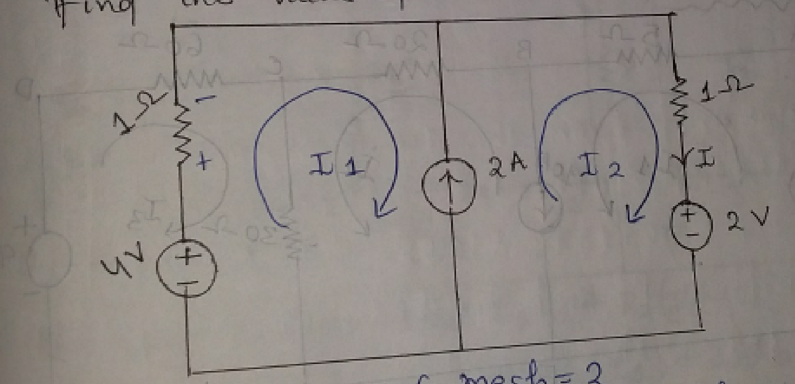
$I_1 = 4A$

$I_2 = 1A$

$I_3 = 1A$

Example :-

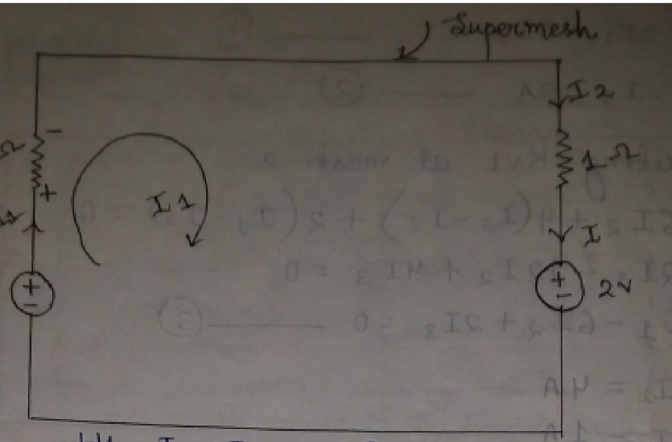
Find the value of current I.



Solution :- (i) no. of mesh = 2

(ii) ✓ clockwise direction

(iii) mesh 1 : $= +4 - I_1$?



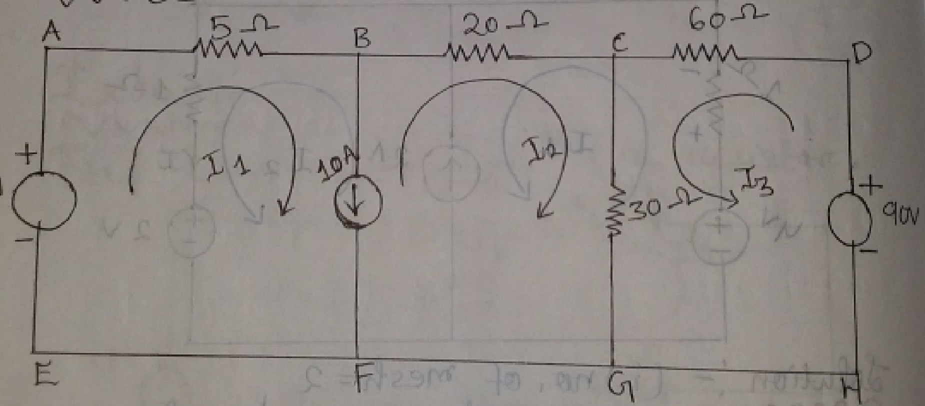
$$+4 - I_1 - I_2 - 2 = 0$$

$$I_1 + I_2 = 2 \quad \text{——— (1)}$$

$$I_2 - I_1 = 2 \quad \text{——— (2)}$$

$\left.\right\} I_2 = I = 2A$ (ans)

Problem :—



Find current in 20Ω by supermesh method.

Solution :— In mesh 3 :—

$$90 = 60I_3 + 30(I_3 + I_2)$$

$$90 = 90I_3 + 30I_2 \quad \text{——— (1)}$$

In mesh 1 and 2 :-

$I_1 - I_2 - 10 = 0$

$I_1 - I_2 = 10$ —— (2)

$30 = 5I_1 + 20I_2 + 30(I_2 + I_3)$

$30 = 5I_1 + 50I_2 + 30I_3$ —— (3)

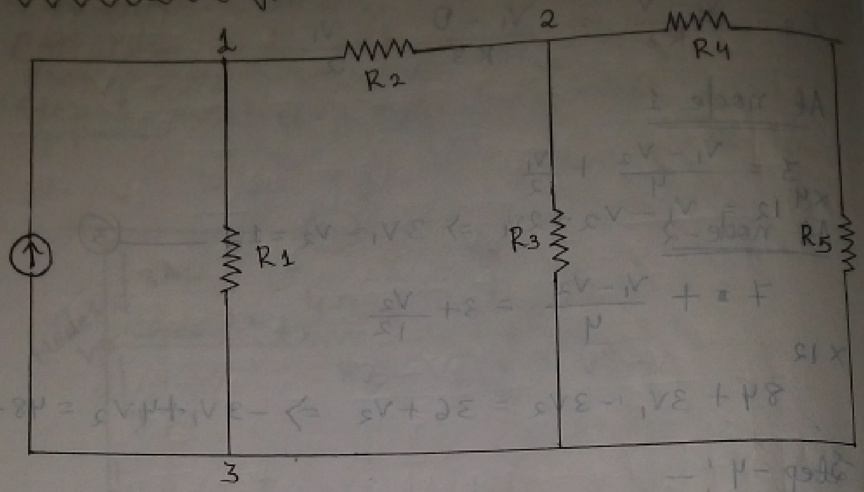
$I_1 = 8.889\,A$

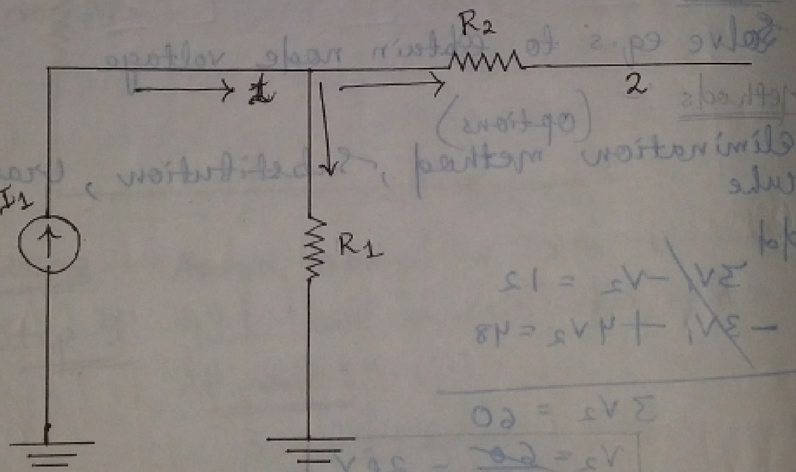
$I_2 = -1.11\,A$

$I_3 = -1.37\,A$

$I_{20\Omega} = -1.11\,A$

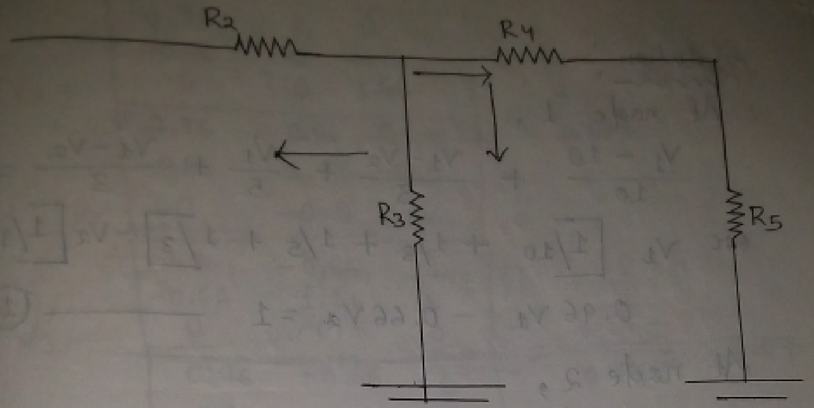
Nodal analysis :—





$$I_1 = V_1/R_1 + (V_1 - V_2)/R_2$$

where V_1 and V_2 are the voltages at node 1 and 2 respectively.



$(V_2 - V_1)/R_2 + V_2/R_3 + V_2/(R_4 + R_5) = 0$

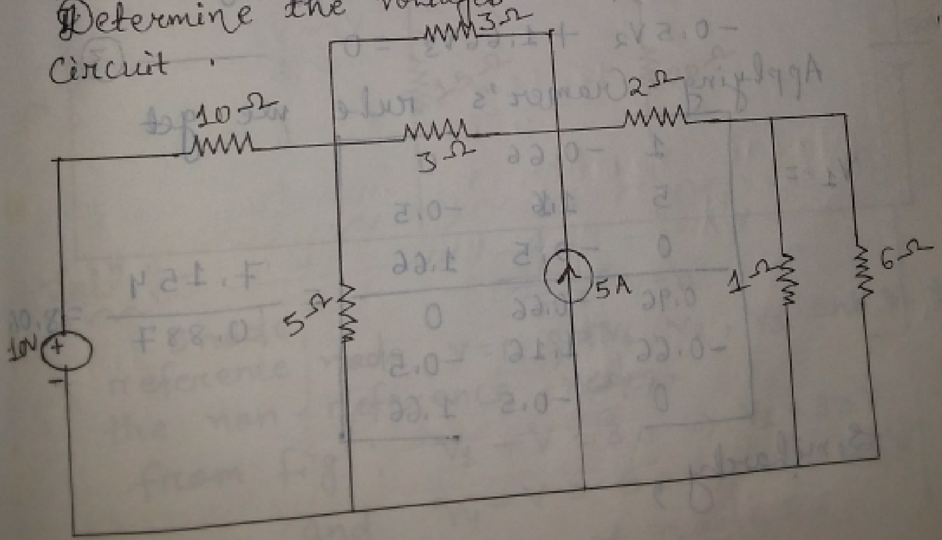
rearranging the above equations, we have

$V_1 \left[1/R_1 + 1/R_2\right] - V_2(1/R_2) = I_1$

$-V_1(1/R_2) + V_2\left[1/R_2 + 1/R_3 + 1/(R_4+R_5)\right] = 0$

Problem :-

(1) Determine the voltages at each node for the circuit.



Solution :-

At node 1,

$$\frac{V_1 - 10}{10} + \frac{V_1 - V_2}{3} + \frac{V_1}{5} + \frac{V_1 - V_2}{3} = 0$$

or $V_1 \left[1/10 + 1/3 + 1/5 + 1/3\right] - V_2 \left[1/3 + 1/3\right]$

$$0.96 V_1 - 0.66 V_2 = 1 \quad \text{——— (1)}$$

At node 2,

$$\frac{V_2 - V_1}{3} + \frac{V_2 - V_1}{3} + \frac{V_2 - V_3}{2} = 5$$

$$-V_1 \left[2/3\right] + V_2 \left[1/3 + 1/3 + 1/2\right] - V_3 \left[1/2\right] = 5$$

$$-0.66 V_1 + 1.16 V_2 - 0.5 V_3 = 5 \quad \text{——— (2)}$$

At node 3

$$(V_3 - V_2)/2 + V_3/I + V_3/6 = 0$$

$$-0.5 V_2 + 1.66 V_3 = 0 \quad \text{——— (3)}$$

Applying Cramer's rule we get

$$V_1 = \frac{\begin{vmatrix} 1 & -0.66 & 0 \\ 5 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}} = \frac{7.154}{0.887} = 8.06$$

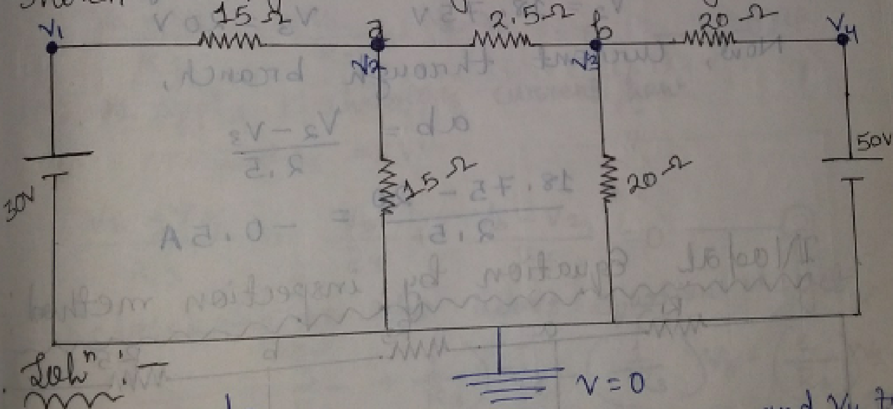
Similarly,

$$V_2 = \frac{\begin{vmatrix} 0.96 & 1 & 0 \\ -0.66 & 5 & -0.5 \\ 0 & 0 & 1.66 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}} = \frac{9.06}{0.887} = 10.2$$

$$V_3 = \frac{\begin{vmatrix} 0.96 & -0.66 & 1 \\ -0.66 & 1.16 & 5 \\ 0 & -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}} = \frac{2.73}{0.887} = 3.07$$

Problem :—

Find current through branch ab of circuit shown in figure using nodal analysis.



Solⁿ :—

5node

reference node $V = 0$ and V_1, V_2, V_3 and V_4 to the non-reference nodes.

from fig : $V_1 - V = 30 \Rightarrow V_1 = 30\,V$

and $V_4 - V = 50 \Rightarrow V_4 = 50\,V$

Now applying KCL at node 2,

$$\frac{V_2 - V_1}{15} + \frac{V_2 - V_3}{2.5} + \frac{V_2 - V}{15} = 0$$

$\Rightarrow V_2 - V_1 + 6(V_2 - V_3) + V_2 - V = 0$

$\Rightarrow V_2 - V_1 + 6V_2 - 6V_3 + V_2 = 0$

$\Rightarrow 8V_2 - 6V_2 = 30$ —— ① $\boxed{\because V_1 = 30V}$

Similarly applying KCL at node 3,

$$\frac{V_3 - V_2}{2.5} + \frac{V_3 - V_4}{20} + \frac{V_3 - V}{20} = 0$$

$\Rightarrow 8(V_3 - V_2) + V_3 - V_4 + V_3 - V = 0$

$\Rightarrow 8V_3 - 8V_2 + V_3 - V_4 + V_3 = 0$

$\Rightarrow -8V_2 + 10V_3 = 50$ —— ② $\boxed{\because V_4 = 50V}$

Solving equⁿ ① and ②, we get

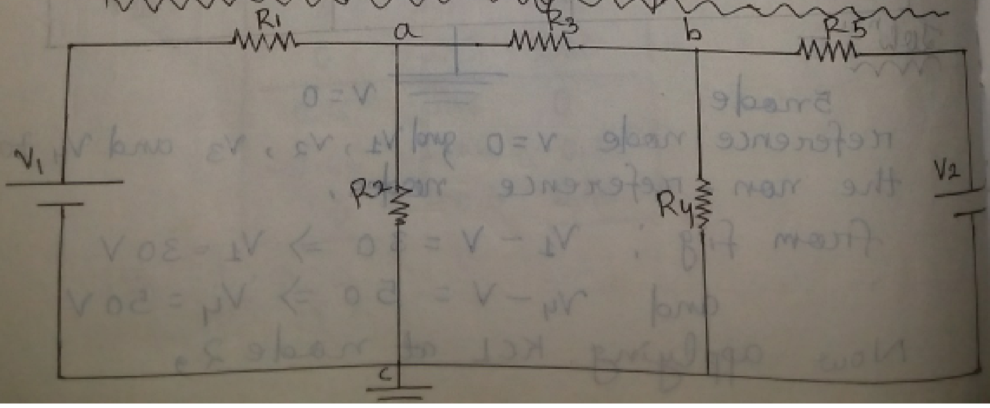
$V_2 = 18.75V$, $V_3 = 20V$

Now, current through branch,

$$ab = \frac{V_2 - V_3}{2.5}$$

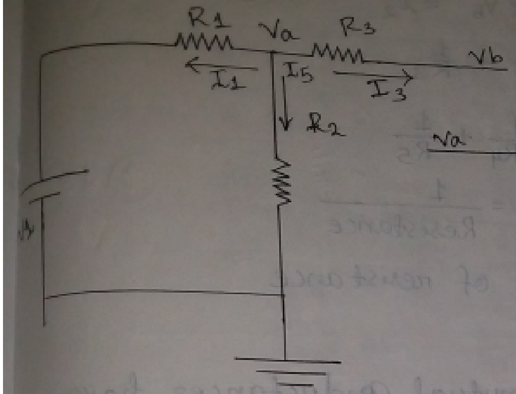
$$= \frac{18.75 - 20}{2.5} = -0.5A$$

Nodal Equation by inspection method :-

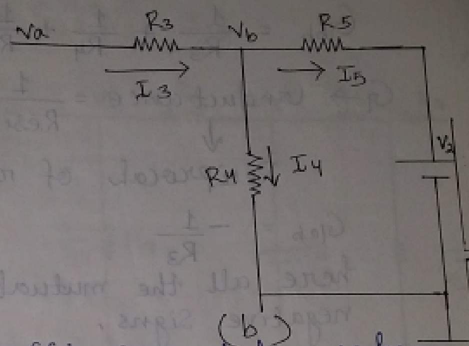


nodes = a, b

reference node = c



(a)



(b)

(a) According to Kirchhoff's current law we have

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} = 0 \quad \text{——} (1)$$

(b) we apply Kirchhoff's current law

$$I_4 + I_5 = I_3$$

$$\frac{V_b - V_a}{R_3} + \frac{V_b}{R_4} + \frac{V_b - V_2}{R_5} = 0 \quad \text{——} (2)$$

Rearranging the above equations we get

$$\underbrace{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}_{G_{aa}} V_a - \underbrace{\left(\frac{1}{R_3}\right)}_{G_{ab}} V_b = \left(\frac{1}{R_1}\right) V_1 \quad \text{——} (3)$$

$$\left(-\frac{1}{R_3}\right) V_a + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) V_b = \frac{V_2}{R_5} \quad \text{——} (4)$$

In general, the above equation can be written as

$$G_{aa} V_a + G_{ab} V_b = I_1 \quad \text{—— (5)}$$
$$G_{ba} V_a + G_{bb} V_b = I_2 \quad \text{—— (6)}$$

$$G_{aa} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$G_{bb} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}$$

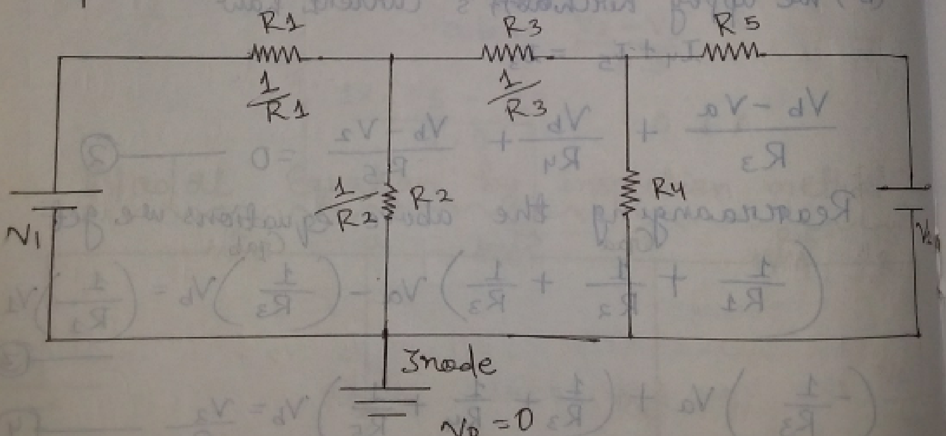
$G \rightarrow$ Conductance $= \frac{1}{\text{Resistance}}$
↓
Reciprocal of resistance

$$G_{ab} = -\frac{1}{R_3}$$

here all the mutual conductances have negative signs.

$$G_{ba} = -\frac{1}{R_3}$$

G_{aa} = Sum of self conductance of node 1.
G_{ab} = mutual conductance between node 1 and 2.



G_{ab} = mutual conductance between node 1 and 2. $\left(\frac{1}{R_3}\right)$

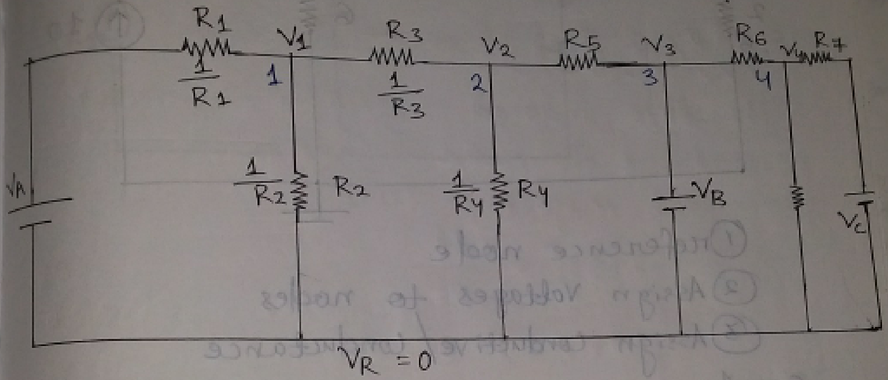
$$G_{ba} = \frac{1}{R_3}$$

$$G_{bb} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}$$

$$G_{11} V_1 \pm G_{12} V_2 \pm G_{13} V_3 = I_1$$

$$\pm G_{21} V_1 + G_{22} V_2 \pm G_{23} V_3 = I_2$$

$$\pm G_{31} V_1 \pm G_{32} V_2 + G_{33} V_3 = I_3$$



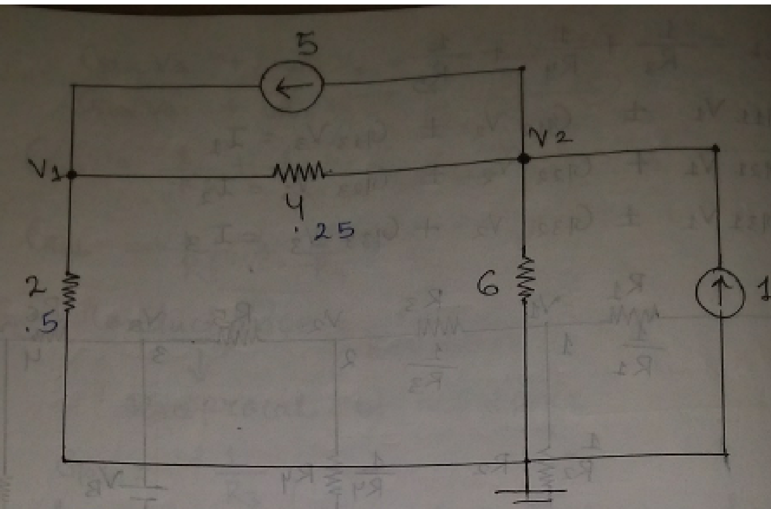
$$G_{11} V_1 \pm G_{12} V_2 \pm G_{13} V_3 \pm G_{14} V_4 = I_1$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V_1 \pm \frac{1}{R_3} V_2 + 0 + 0 = \frac{V_A}{R_2}$$

$$\pm G_{21} V_1 + G_{22} V_2 \pm G_{23} V_3 \pm G_{24} V_4 = I_2$$

$$\pm G_{31} V_1 \pm G_{32} V_2 \pm G_{33} V_3 \pm G_{34} V_4 = I_2$$

$$\pm G_{41} V_1 \pm G_{42} V_2 \pm G_{43} V_3 + G_{44} V_4 = I_3$$



① reference node
② Assign voltages to nodes
③ Assign conductive/conductance

$G = \frac{1}{R}$

Nodal analysis by inspection method :

① Find a reference node with known voltage
② Assign voltages to other nodes
③ Assign conductance to each resistor
④ Find the elements of the conductance matrix
ⓐ For each node G_{ii}
ⓑ between each 2 nodes $G_{ij} = G_{ji}$
⑤ Set up the node equations in matrix form
⑥ Solve the node-voltage equations.

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